

Energy Technology Systems Analysis Programme  
TIMES Version 2.0 User Note

# TIMES Damage functions

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## Foreword

This report contains the full documentation on the implementation and usage of the new damage cost functions for the TIMES model.

The report is divided in four chapters. Chapter 1 contains an introduction, and Chapter 2 presents a brief description of the mathematical approach taken. Chapter 3 contains the description of the GAMS implementation of the new elements, along with the sets, parameters, variables, and equations that have been added to the TIMES model. Finally, Chapter 4 constitutes a brief User's Manual for the damage functions in TIMES.

This documentation may eventually also be inserted in the complete documentation of the TIMES model.

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# 1. INTRODUCTION

The Damage Cost Function option of TIMES is intended for modelers who wish to evaluate the environmental externalities caused by an energy system. For instance, emissions of toxic or environmentally harmful pollutants from the energy system create social costs linked to impacts of the pollution on human health and the environment. In another example, in global studies of GHG emissions, it may be of interest to evaluate the impact of GHG emissions on concentrations and ultimately on damages created by climate change induced by increased concentration of GHG's.

Until recently, in most studies involving bottom-up models emission externalities have been modeled in one of two ways: either by introducing an emission tax, or by imposing emission caps. In the first case, the tax is (ideally) supposed to represent the external cost created by one unit of emission. However, using a tax assumes that the cost is a linear function of emissions. In the second approach, it is assumed that such a cost is unknown but that exogenous studies (or regulations, treaties, etc.) have defined a level of acceptable emissions that should not be exceeded. However, using this approach is akin to making the implicit assumption that emissions in excess of the cap have an infinite external cost. Both of these approaches have merit and have been successfully applied to many energy system model studies.

It is however possible to extend these two approaches by introducing an option to better model the cost of damages created by emissions. The damage function option discussed in this document extends the concept of an emission tax by modeling more accurately the assumed cost of damages due to emissions of a pollutant.

## 2. MATHEMATICAL FORMULATION

### 2.1 General

With respect to optimization, two distinct approaches to account for damage costs can be distinguished:

1. Environmental damages are computed ex-post, without feedback into the optimization process, and
2. Environmental damages are part of the objective function and therefore taken into account in the optimization process.

In both approaches, a number of assumptions are made:

- Emissions in each region may be assumed to cause damage only in the same region or, due to transboundary pollution, also in other regions; however, all damage costs are allocated to the polluters in the source region, in accordance with the Polluter Pays Principle, or Extended Polluter Responsibility;
- Damages in a given time period are linked to emissions in that same period only (damages are not delayed, nor are they cumulative); and
- Damages due to several pollutants are the sum of damages due to each pollutant (no cross impacts).

In a given time period, and for a given pollutant, the damage cost is modeled as follows:

$$DAM(EM) = \alpha \cdot EM^{\beta+1} \quad (1)$$

where:

- EM is the emission in the current period;
- DAM is the damage cost in the current period;
- $\beta \geq 0$  is the elasticity of marginal damage cost to amount of emissions; and
- $\alpha > 0$  is a calibrating parameter, which may be obtained from dose-response studies that allow the computation of the marginal damage cost per unit of emission at some reference level of emissions.

If we denote the marginal cost at the reference level  $MC_0$ , the following holds:

$$MC_0 = \alpha \cdot (\beta + 1) \cdot EM_0^\beta \quad (2)$$

where  $EM_0$  is the reference amount of emissions. Therefore expression (1) may be re-written as:

$$DAM(EM) = MC_0 \cdot \frac{EM^{\beta+1}}{(\beta+1) \cdot EM_0^\beta} \quad (3)$$

The marginal damage cost is therefore given by the following expression:

$$MC(EM) = MC_0 \cdot \frac{EM^\beta}{EM_0^\beta} \quad (4)$$

The approach to damage costs described in this section applies more particularly to local pollutants. Extension to global emissions such as GHG emissions requires the use of a global TIMES model and a reinterpretation of the equations discussed above.

The modeling of damage costs via equation (3) introduces a non-linear term in the objective function if the  $\beta$  parameter is strictly larger than zero. This in turn requires that the model be solved via a Non-Linear Programming (NLP) algorithm rather than a LP algorithm. However, the resulting Non-Linear Program remains convex as long as the elasticity parameter is equal to or larger than zero. For additional details on convex programming, see Nemhauser et al (1989). If linearity is desired (for instance if problem instances are very large), we can approximate expression (3) by a sequence of linear segments with increasing slopes, and thus obtain a Linear Program.

The linearization can be done, by choosing a suitable range of emissions, and dividing that range into  $m$  intervals below the reference level, and  $n$  intervals above the reference level. We also assume a middle interval centered at the reference emission level. To each interval corresponds one step variable  $S$ . Thus, we have for emissions:

$$EM = \sum_{i=1}^m S_i^{lo} + S^{mid} + \sum_{i=1}^n S_i^{up} \quad (5)$$

The damage cost can then be written as follows:

$$DAM(EM) = \sum_{i=1}^m MC_i^{lo} \cdot S_i^{lo} + MC_0 \cdot S^{mid} + \sum_{i=1}^n MC_i^{up} \cdot S_i^{up} \quad (6)$$

where:

- $MC_i^{lo}$  and  $MC_i^{up}$  are the approximate marginal costs at each step below and above the reference level as shown in (7) below; and
- $S_i^{lo}$ ,  $S^{mid}$  and  $S_i^{up}$  are the non-negative step variables for emissions. Apart from the final step, each step variable has an upper bound equal to the width of the interval. In this formulation we choose intervals of uniform width on each side of the reference level. However, the intervals below and above the reference level

can have different sizes. The width of the middle interval is always the average of the widths below and above the reference level.

The approximate marginal costs at each step can be assumed to be the marginal costs at the center of each step. If all the steps intervals are of equal size, the marginal costs for the steps below the reference level are obtained by the following formula:

$$MC_i^{lo} = MC_0 \cdot \left( \frac{(i - 0.5)}{(m + 0.5)} \right)^\beta \quad (7)$$

Formulas for the marginal costs of the other steps can be derived similarly.

The TIMES implementation basically follows the equations shown above. Both the non-linear and linearized approach can be used. However, in order to provide some additional flexibility, the implementation supports also defining a threshold level of emissions, below which the damage costs are zero. This refinement can be taken into account in the balance equation (5) by adding one additional step variable having an upper bound equal to the threshold level, and by adjusting the widths of the other steps accordingly. The threshold level can also easily be taken into account in the formulas for the approximate marginal costs.

In addition, the implementation supports different elasticities and step sizes to be used below and above the reference level. See Section 3 for more details.

## 3. GAMS IMPLEMENTATION

### 3.1 Overview

As discussed in Section 2, TIMES has facilities to permit the assessment of environmental externalities by means of two approaches to determine the impact or cost of damages arising from emissions. The second approach can be further divided into the non-linear and linear formulations, and therefore the following three approaches are available in Standard TIMES:

1. The environmental damages are computed ex-post (\$SET DAMAGE NO), without feedback into the optimization process,
2. The environmental damages are a linearized part of the objective function (\$SET DAMAGE 'LP') and therefore taken into account in the optimization process;
3. The environmental damages are a non-linear part of the objective function (\$SET DAMAGE 'NLP') and therefore taken into account in the optimization process.

In order to model environmental damages in TIMES, the standard model formulation is in the first case essentially untouched, and in the latter two cases it has an augmented objective function, which is either linear or non-linear. The data requirements and model adjustments related to the damage options are presented in the rest of this section. The standard TIMES result parameters include entries added to report on the period-wise damage costs.

Note that owing to the non-linear nature of the modified objective function that endogenizes the damages, the NLP damage option requires non-linear solution methods that can lead to much larger resource utilization compared to LP models. In addition, the active DAMAGE options with augmented objective function cannot be currently activated with the TIMES-MACRO model variant, as presented in Table 1 below.

*Table 1. Valid combinations of TIMES-Damage with other TIMES model variants.*

TIMES Variant	LP	NLP
Standard TIMES	YES	YES
Stochastic TIMES	YES	YES
TIMES-ETL/DSC	YES (MIP)	YES (MINLP)
TIMES-MACRO	Not implemented	Not implemented

## 3.2 Parameters

### 3.2.1 Input parameters

All the parameters for describing damage functions are available in the VEDA-FE shell, where they may be specified. All parameters have a prefix 'DAM\_' in the GAMS code of the model generator. The parameters are discussed in more detail below:

1. The parameter **DAM\_COST** is used to specify the marginal damage cost at the reference level of emissions. The parameter has a year index, which can be utilized also for turning damage accounting on/off for an emission in a period (by specifying an EPS value for the cost). **DAM\_COST** is interpolated/extrapolated by default, but unlike other cost parameters, the interpolation is sparse, and the costs are assumed to be constant within each period.
2. The parameter **DAM\_BQTY** is used to specify the reference level of emissions. If not specified or set to zero, the marginal damage costs will be assumed constant, and no emission steps are used.
3. The parameter **DAM\_ELAST** is used to specify the elasticity of marginal damage costs to emissions in the lower and upper direction. If specified in one direction only, the elasticity is assumed in both directions. If neither is specified, the marginal damage costs will be constant in both directions.
4. The parameter **DAM\_STEP** can be used for specifying the number of emission steps below and above the reference level of emissions. The last step above the reference level will always have an infinite bound. If the number of steps is not provided in either direction, but the elasticity is, one step is assumed in that direction. If the NLP formulation is used (**DAMAGE==NLP**), all **DAM\_STEP** parameters will be ignored.
5. The parameter **DAM\_VOC** can be used for specifying the variation in emissions covered by the emission steps, both in the lower an upper direction. The variation in the lower direction should be less than or equal to the reference level of emissions. If the lower variation is smaller than **DAM\_BQTY**, the damage costs

*Table 2. Input parameters for the TIMES Damage cost functions.*

Parameter	Description
DAM_COST(r,y,c,cur)	Marginal damage cost at reference emission level
DAM_BQTY(r,c)	Reference emission level
DAM_ELAST(r,c,bd)	Elasticity of marginal damage cost on the lower and upper side
DAM_STEP(r,c,bd)	Number of steps for the linearized damage cost function
DAM_VOC(r,c,bd)	Variation of stepped emissions on the lower and upper side

are zero for emissions below the difference. The lower variance can thus be used for defining a threshold level for the damage costs. If `DAM_VOC` is not specified in the lower direction, it is assumed to be equal to `DAM_BQTY`. If `DAM_VOC` is not specified in the upper direction, the emission step size in the upper direction is assumed to be equal to that in the lower direction. If the NLP formulation is used (`DAMAGE==NLP`), any `DAM_VOC` parameters specified in the upper direction will be ignored. However, even in the NLP formulation the lower `DAM_VOC` can be used for defining a threshold emission level for the costs.

### 3.2.2 Reporting parameters

There is only one reporting parameter related to the Damage Cost functions. The parameter represents the undiscounted damage costs by region, period and emission commodity. The parameter has two flavors; the first one is for standard TIMES and the second one for stochastic TIMES:

- `CST_DAM(r,t,c)`:  
Annual damage costs from emission C in region R,
- `SCST_DAM(w,r,t,c)`:  
Annual damage costs from emission C in region R and scenario W.

These parameters have been included in the updated `.vdd` files that describe the parameters to be transferred to VEDA-BE under standard TIMES and stochastic TIMES. Therefore, the corresponding result parameter is always available in VEDA-BE whenever Damage Cost functions have been defined, even with the setting `DAMAGE==NO`.

The damage costs are always reported by using the accurate non-linear expressions, even if the linearized formulation is chosen for the augmented objective function.

### 3.3 Variables

There is only two sets of new variables in the damage cost formulation, VAR\_DAM and VAR\_OBJDAM which are shown below in Table 3. The variables VAR\_DAM represent the steps in the emissions in each period. In the linearized formulation, there are DAM\_STEP(...,'LO') number of step variables on the lower side and DAM\_STEP(...,'UP') number of step variables on the higher side of emissions. In addition, one step variable of type 'FX' corresponds to the middle step that includes the reference level of emissions, and an optional additional step variable of type 'FX' corresponds to the zero-damage fraction of emissions, as defined by the difference between DAM\_BQTY(..) and DAM\_VOC(...,'LO').

The variables VAR\_OBJDAM represent the total discounted damage costs by region. The undiscounted costs in each period described in Section 2 are discounted and summed over all periods and emissions in each region. As emissions are in TIMES assumed to be constant within each period, damage costs are likewise assumed to be constant within each period.

As there is thus essentially no other changes to the model variables, the user is referred to Chapter 4 of the TIMES Reference Manual for details on the variables of the model.

### 3.4 Equations

Below in Table 4 the few equations related to the damage functions version are listed and briefly described. The equations include the balance of stepped emissions, the objective component for damage costs, and the augmented total objective function.

Note that the equation for the augmented final objective function, EQ\_OBJ, has the same name as in standard TIMES, only the definition of this equation is different when damage cost functions are activated. Similarly, also the objective variable ObjZ has the same name as in standard TIMES.

*Table 3. Variables for damage functions in TIMES.*

Variable	Description
VAR_DAM(r,t,c,bd,j)	The emission step variable for damage functions.
VAR_OBJDAM(r,cur)	The variable equal to the sum of the total discounted damage costs in each region.

Table 4. Equations for damage functions in TIMES.

Equation	Description
EQ_DAMAGE(r,t,c)	The balance equation between the stepped emission variables and the total emissions in each period.
EQ_OBJDAM(r,cur)	The total discounted damage costs by region, which will be added as a component to the objective function.
EQ_OBJ	The augmented objective function, i.e. the full objective function with an additional term describing the discounted damage costs.

### 3.5 Changes in model generator code

The implementation required only small modifications to the existing code and only two new components in the model generator code. The new and modified code components are listed in Table 5.

The file EQDAMAGE.mod is automatically called from the file EQMAIN.mod, if the parameter DAM\_COST has been defined. Similarly, the variable VAR\_OBJDAM is conditionally included in the objective in the file EQOBJ.mod, and the new equations are included in the MODEL statement. Finally, the reporting file RTP\_DAM.mod is called from the file RPTMAIN.mod.

Table 5. New and modified files in the TIMES model generator code.

Added file	Description
EQDAMAGE.mod	Preprocessing of damages and damage-related equations
RPT_DAM.mod	Calculation of the reporting parameter of damage costs
Modified file	Description of changes made
INITMTY.mod	Declaration of the damage input parameters included
EQOMAIN.mod	Preprocessing routine for damage equations conditionally called
EQOBJ.mod	Damage component of the objective function conditionally included
MOD_VARS.mod	Damage variables conditionally included in the model
MOD_EQUA.mod	Damage equations conditionally included in the model
SOLVE.mod	Solution method set to NLP or MINLP when DAMAGE==NLP
ERR_STAT.mod	Locally optimal solve status allowed when checking solution status
RPTMAIN.mod	Reporting routine for damage costs conditionally called

## 4. USER'S REFERENCE

### 4.1 Activating the damage functions

The linearized Damage Cost Functions are by default automatically activated in TIMES whenever the DAM\_COST parameter is specified for at least one emission commodity in some region. However, the activation of the Damage Functions can also be controlled by using the one of the following settings in the run file (the trailing comment part starting from '!' should not be included):

```
$ SET DAMAGE LP      ! Activate linear damage functions
```

```
$ SET DAMAGE NLP     ! Activate non-linear damage functions
```

```
$ SET DAMAGE NO      ! Categorically deactivate damage functions
```

In all cases, the more accurate non-linear damage costs are always reported if the DAM\_COST parameter is specified for at least one emission commodity in some region.

**Remark:** The implementation provides also a quick hack solution for defining a damage cost function for global CO<sub>2</sub> concentration level, if deemed useful. Such a cost function can be defined separately for each world region or for the globe.

### 4.2 Specification of input parameters

The following Table 6 lists the possible user-input parameters. Only the damage cost parameter DAM\_COST has to be provided by the user to include damage costs in the model. However, if the DAM\_BQTY and DAM\_ELAST parameters are not specified, the marginal damage costs will be constant. The following indexes are used in the index domain of the parameters:

- r: region,
- datayear: period/milestoneyear,
- c: emission commodity,
- bd: bound type (LO/UP)
- cur: currency.

**Table 6: Input parameters for TIMES-Damage**

<b>Input parameter (Indexes)<sup>1</sup></b>	<b>Related parameters<sup>2</sup></b>	<b>Units / Ranges &amp; Default values &amp; Default inter-/extrapolation<sup>3</sup></b>	<b>Instances<sup>4</sup></b> (Required / Omit / Special conditions)	<b>Description</b>	<b>Affected equations or variables<sup>5</sup></b>
DAM_COST (r,datayear,c,cur)	DAM_BQTY, DAM_ELAST, DAM_STEP, DAM_VOC	<ul style="list-style-type: none"> <li>• TIMES cost unit</li> <li>• [0, INF); default value: none</li> <li>• Default i/e<sup>6</sup>: standard</li> </ul>	<ul style="list-style-type: none"> <li>• Required for each commodity for which damage costs are to be accounted.</li> </ul>	Marginal damage cost of emission c at reference emission level.	<ul style="list-style-type: none"> <li>• EQ_OBJDAM</li> </ul>
DAM_BQTY (r,c)	See above	<ul style="list-style-type: none"> <li>• TIMES emission unit</li> <li>• [0, INF); default value: 0</li> </ul>	<ul style="list-style-type: none"> <li>• Only taken into account if DAM_COST has been specified</li> </ul>	Reference level of emissions c	<ul style="list-style-type: none"> <li>• EQ_DAMAGE</li> <li>• EQ_OBJDAM</li> </ul>
DAM_ELAST (r,c,bd)	See above	<ul style="list-style-type: none"> <li>• Dimensionless</li> <li>• [0, INF); default value: 0</li> </ul>	<ul style="list-style-type: none"> <li>• Only taken into account if DAM_COST has been specified</li> </ul>	Elasticity of marginal damage cost to emissions	<ul style="list-style-type: none"> <li>• EQ_OBJDAM</li> </ul>
DAM_STEP (r,c,bd)	See above	<ul style="list-style-type: none"> <li>• Dimensionless</li> <li>• [0, INF), integer; default value: 0</li> </ul>	<ul style="list-style-type: none"> <li>• Only taken into account if DAM_COST has been specified</li> </ul>	Number of emission steps in the lower/upper direction	<ul style="list-style-type: none"> <li>• EQ_DAMAGE</li> <li>• EQ_OBJDAM</li> </ul>
DAM_VOC (r,c,bd)	See above	<ul style="list-style-type: none"> <li>• TIMES emission unit</li> <li>• (0, INF); <math>\leq</math> DAM_BQTY; default value: DAM_BQTY</li> </ul>	<ul style="list-style-type: none"> <li>• Only taken into account if DAM_COST has been specified</li> </ul>	Variation in emissions covered by the emission steps in the lower/upper direction. A threshold emission level can be defined with bd='LO'.	<ul style="list-style-type: none"> <li>• EQ_DAMAGE</li> <li>• EQ_OBJDAM</li> </ul>

<sup>1</sup> The first row contains the parameter name, the second row contains in brackets the index domain over which the parameter is defined.

<sup>2</sup> This column gives references to related input parameters or sets being used in the context of this parameter as well as internal parameters/sets or result parameters being derived from the input parameter.

<sup>3</sup> This column lists the unit of the parameter, the possible range of its numeric value [in square brackets] and the inter-/extrapolation rules that apply.

<sup>4</sup> An indication of circumstances for which the parameter is to be provided or omitted, as well as description of inheritance/aggregation rules applied to parameters having the timeslice (s) index.

<sup>5</sup> Equations or variables that are directly affected by the parameter.

<sup>6</sup> Abbreviation i/e = inter-/extrapolation

### 4.3 Examples

Assume that we wish to define linearized damage costs for the emission commodity 'EM' so that the cost function has the following properties:

- The reference level of emissions is 80 units;
- The marginal cost at the reference level are 10 cost units per emission unit;
- The cost elasticity is 1 in the lower direction, and 0.7 in the upper direction;

The damage function can be specified with the following parameters:

```
PARAMETER DAM_COST / REG. 2000. EM. CUR 10 /;  
PARAMETER DAM_BQTY / REG. EM 80 /;  
PARAMETER DAM_ELAST / REG. EM. LO 1, REG. EM. UP 0.7 /;
```

As we did not specify the number of steps, but we did specify the elasticities in both directions, the number of steps is assumed to be 1 in both directions. The resulting damage cost function is illustrated in Figure 1. Because the damage function has a very coarse representation, the total costs have notable deviations from the accurate non-linear function. Note that the step size has been automatically determined to be  $DAM\_BQTY / (DAM\_STEP + 0.5) = 80 / 1.5$ . However, the last step has no upper bound.

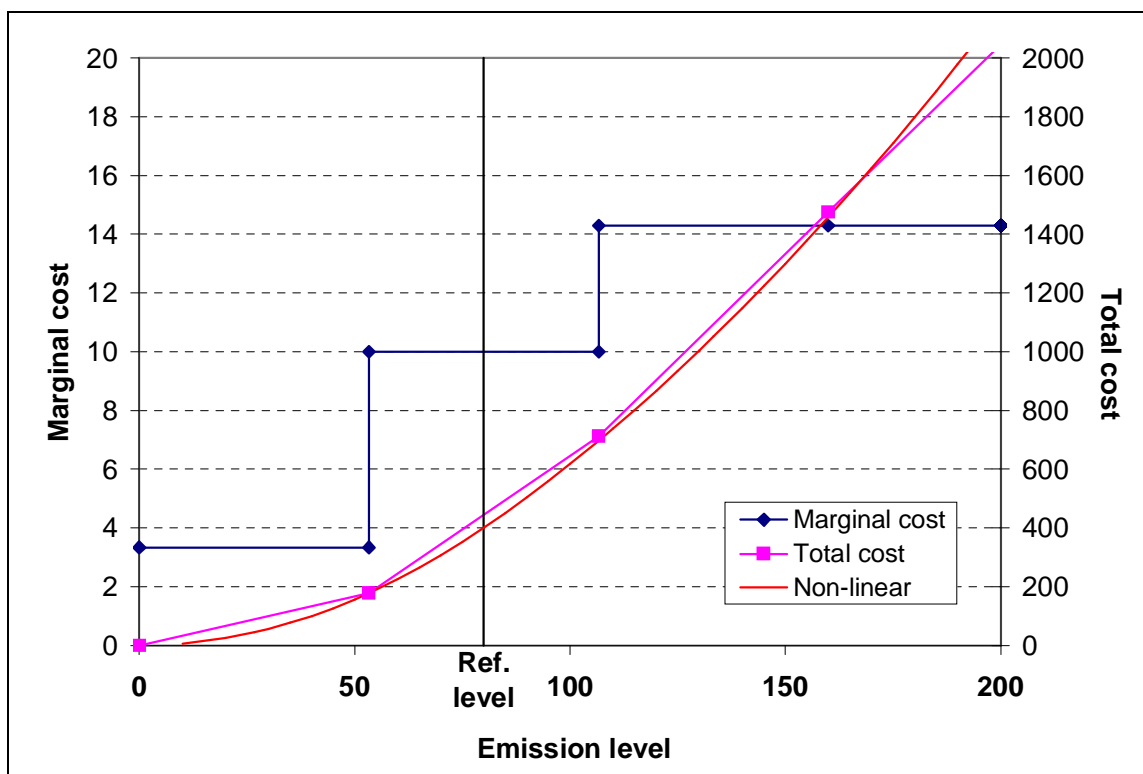


Figure 1. Example of a linearized damage function with 1+1+1 steps (1 lower step, middle step, 1 upper step).

Assume next that we would like to refine the damage function by the following specifications:

- We want to have 5 steps below the reference, and 3 steps above it;
- The threshold level of damage costs is 20 units of emissions;
- The steps above the reference level should cover 100 units of emissions.

The damage function can be specified with the following parameters

```

PARAMETER DAM_COST / REG. 2000. EM. CUR 10 /;
PARAMETER DAM_BQTY / REG. EM 80 /;
PARAMETER DAM_ELAST / REG. EM. LO 1, REG. EM. UP 0.7 /;
PARAMETER DAM_STEP / REG. EM. LO 5, REG. EM. UP 3 /;
PARAMETER DAM_VOC / REG. EM. LO 60, REG. EM. UP 100 /;

```

The resulting damage cost function is illustrated in Figure 2. The cost function follows now very closely the accurate non-linear function. Note that the step sizes derived from the VOC specifications are 10 units for the lower steps, 20 for the middle step, and 30 units for the upper steps. However, the last step of course has no upper bound.

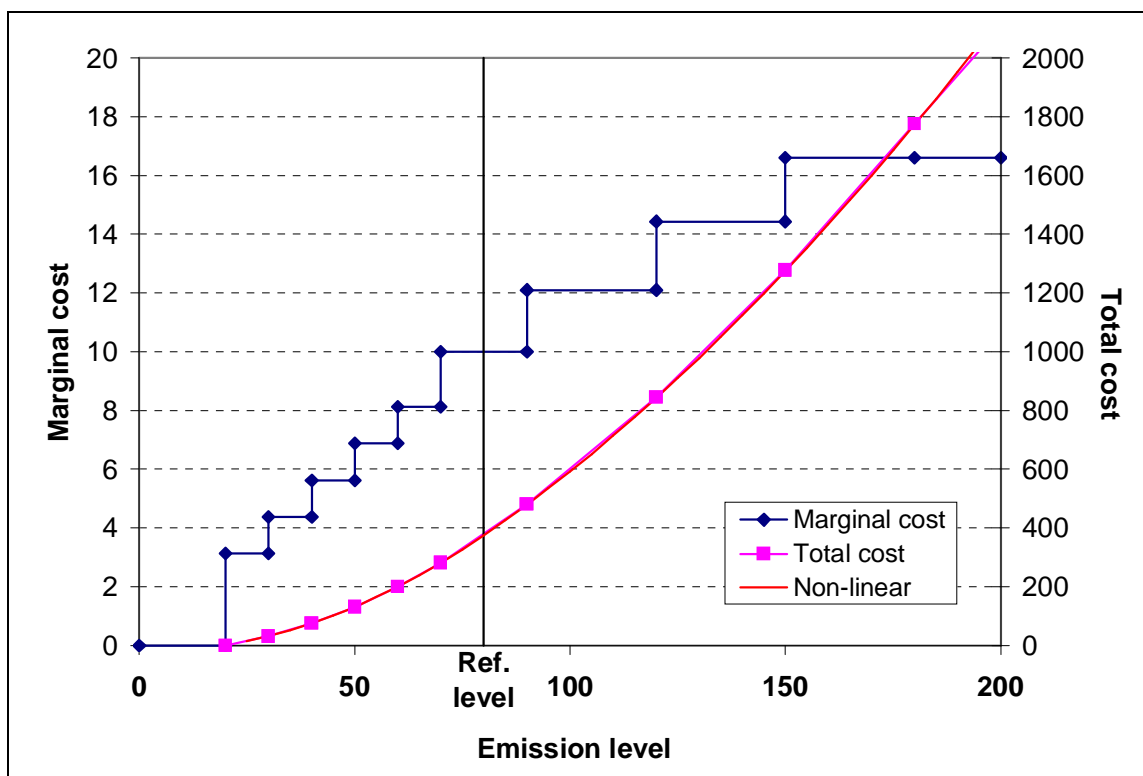


Figure 2. Example of a linearized damage function with 1+5+1+3 steps (zero cost step, 5 lower steps, middle step, 3 upper steps).

#### 4.4 Exporting results to VEDA-BE

For reporting the damage cost results, the attributes listed in Table 7 have been added for the transfer of results into VEDA\_BE:

Since these attributes are defined by the model even when the damage functions are not used, the standard vdd file (times2veda.vdd) can be applied when transferring results from TIMES to VEDA-BE by using the gdx2veda utility. If damage functions are defined, the costs are reported also in the case where the damage costs are not part of the objective function (DAMAGE==NO). The user can then choose to either include or exclude these costs in the result analysis.

Table 7. Reporting parameters for the TIMES Damage cost functions.

Parameter	Description
CST_DAM(r,t,c)	Damage costs by region, period and emission (standard TIMES)
SCST_DAM(w,r,t,c)	Damage costs by region, period and emission (stochastic TIMES)

## 5. REFERENCES

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