Energy Technology Systems Analysis Programme

TIMES Version 2.0 User Note

TIMES Climate Module

Authors:

Richard Loulou
KANLO Consultants, France

Maryse Labriet
KANLO Consultants, France

Antti Lehtila
VTT, Finland

Contents

7 The TIMES Climate Module

This chapter contains the full documentation on the Climate Module option for the TIMES model. The chapter is divided in 8 sections: section 7.1 contains a detailed description of the theoretical approach taken, sections 7.2 to 7.6 present the parameters, variables, and equations required by the Climate Module. Section 7.7 discusses the GAMS implementation of the Climate Option, and section 7.8 gives useful references for the chapter's material.

7.1 Formulation of the TIMES Climate Module

7.1.1 Approach taken

The Climate Module starts from global emissions as generated by the TIMES global model, and proceeds to compute successively:

- the changes in CO2 concentrations in three reservoirs,
- the total change (over pre-industrial times) in atmospheric radiative forcing from anthropogenic causes, and
- the temperature changes (over pre-industrial times) in two reservoirs.

The Climate Equations used to perform these calculations are adapted from Nordhaus and Boyer (1999), who proposed linear recursive equations for calculating concentrations and temperature changes. These linear equations give results that are good approximations of those obtained from more complex climate models (Drouet *et al.*, 2004; Nordhaus and Boyer, 1999). In addition, the non-linear radiative forcing equation used by these authors is the same as the one used in most models. The choice of the Nordhaus and Boyer's climate equations is motivated by the simplicity of their approach and by the fact that their climate module is well-documented and acceptably accurate. In our implementation, the forcing equation has been replaced by a linear approximation whose values closely approach the exact ones as long as the useful range is carefully selected.

Rigorously, the concentration and forcing equations used in the climate module are applicable only to the carbon cycle, and a different treatment of other greenhouse gases -- methane, N2O, ozone, aerosols, etc. could be done using specific models of their own life cycles. However, following a commonly accepted approach, it is possible to use the CO2 equations to calculate the impact of other gases on climate. To do so, it is necessary to first convert the emissions of each GHG into a CO2-equivalent quantity, and to add these CO2-equivalents to form a fictitious emission of *total CO2-equivalent*, which is then treated as if it were real CO2 emissions. The coefficients used for converting emissions of other gases into CO2-equivalents are the Global Warming Potentials (GWP) recommended by the IPCC Third Assessment Report (IPCC 2001). Therefore, in what follows, the term CO2 used in the climate equations should really be thought of as CO2-equivalent.

We now describe the mathematical equations used at each of the three steps of the climate module.

7.1.1.1 Concentrations (accumulation of CO2)

CO2 accumulation is represented as the linear three-reservoir model below: the atmosphere, the quickly mixing upper ocean + biosphere, and the deep ocean. CO2 flows in both directions between adjacent reservoirs. The 3-reservoir model is represented by the following 3 equations when the step of the recursion is equal to one year:

$$M_{atm}(y) = E(y-1) + (1 - \varphi_{atm\text{-}up})\, M_{atm}(y-1) + \varphi_{up\text{-}atm}\, M_{up}(y-1) \qquad (1)$$
$$M_{up}(y) = (1 - \varphi_{up\text{-}atm} - \varphi_{up\text{-}lo})\, M_{up}(y-1) + \varphi_{atm\text{-}up}\, M_{atm}(y-1) + \varphi_{lo\text{-}up}\, M_{lo}(y-1) \qquad (2)$$
$$M_{lo}(y) = (1 - \varphi_{lo\text{-}up})\, M_{lo}(y-1) + \varphi_{up\text{-}lo}\, M_{up}(y-1) \qquad (3)$$

with

- $M_{atm}(y)$, $M_{up}(y)$, $M_{lo}(y)$: masses of CO_2 in atmosphere, in a quicly mixing reservoir representing the upper level of the ocean and the biosphere, and in deep oceans (GtC), respectively, at period *t* (GtC)
- $E(y-1)$ = CO_2 emissions in previous year (GtC)
- φ_{ij}, transport rate from reservoir *i* to reservoir *j* (*i*, *j* = *atm*, *up*, *lo*) from year *y-1* to *y*

7.1.1.2 Radiative forcing

The relationship between GHG accumulations and increased radiative forcing, $\Delta F(t)$, is derived from empirical measurements and climate models.

$$\Delta F(t) = \gamma * \frac{\ln(M_{atm}(t)/M_0)}{\ln 2} + O(t) \qquad (4)$$

where:

- M_0 (i.e.CO2ATM_PRE_IND) is the pre-industrial (circa 1750) reference atmospheric concentration of CO2 = 596.4 GtC
- γ is the radiative forcing sensitivity to atmospheric CO_2 concentration doubling = 4.1 W/m^2
- *O(t)* (i.e. EXOFORCING(t)), is the increase in total radiative forcing at period *t* relative to pre-industrial level due to anthropogenic GHG's not accounted for in the computation of CO2 emissions. Units = W/m^2. In Nordhaus and Boyer (1999), only emissions of CO2 were explicitly modeled, and therefore O(t) accounted for all other GHG's. In TIMES, N_2O and CH_4 are fully accounted for, but some are not (e.g. CFC's, aerosols, ozone). Therefore, our values for O(t) will differ from those in Nordhaus and Boyer. It is the modeler's responsibility to include in the calculation of O(t) only those gases not included in the CO2-equivalent emissions.

The parameterization of the forcing equation is not controversial and relies on the IPCC Second Assessment Report by Working Group I (1996). The major assumption made in RICE is also made here: a doubling of CO2 concentrations leads to an increase in radiative forcing $\gamma = 4.1\ \mathrm{W/m^2}$. The IPCC Third Assessment Report by Working Group I (2001) provides a slightly smaller value of $3.7\ \mathrm{W/m^2}$ (based on Table 6.2, p.358, chapter 6), and the latter value is the default value adopted in TIMES. Users are free to experiment with other values of the γ parameter.

In TIMES, the logarithmic forcing function is replaced by the linear approximation shown below (equation 4a), in order to preserve the linearity of the TIMES equations. In a previous TIMES version, the logarithmic function was used, and therefore, the forcing and temperature equations were not treated as *bona fide* TIMES equations, but rather as reporting devices. With the linearized forcing, the forcing and temperature equations are regular TIMES equations, allowing a user to put bounds on these quantities.

The linear approximation is obtained as follows:

First, an interval of interest for M must be selected by the user. The interval should be large enough to accommodate the anticipated values of the concentrations, but not so wide as to make the approximation inaccurate. We denote the interval as (M_1,M_2).

Next, the linear forcing equation is taken as the half sum of two linear expressions, which respectively underestimate and overestimate the exact forcing value. The underestimate consists of the chord of the logarithmic curve, whereas the overestimate consists of the tangent to the logarithmic curve that is parallel to the chord. These two estimates are illustrated in Figure 1 where the interval (M_1,M_2) is from 375 ppm to 550 ppm.

By denoting the pre-industrial concentration level as M_0, the general formulas for the two estimates are as follows:

Overestimate:
$$F_1(M) = \frac{\gamma}{\ln 2}\cdot\left[\ln\left(\frac{\gamma}{slope\cdot\ln(2)\cdot M_0}\right) - 1\right] + slope\cdot M \tag{5}$$

Underestimate:
$$F_2(M) = \gamma\cdot\ln(M_1/M_0)/\ln 2 + slope\cdot(M - M_1) \tag{6}$$

Final approximation:
$$F_3(M) = \frac{F_1(M) + F_2(M)}{2} \tag{7}$$

where:
$$slope = \gamma\cdot\frac{\ln(M_2/M_1)/\ln 2}{(M_2 - M_1)}$$

The linearized forcing function implemented in the TIMES model generator is based on the final approximation, which takes the average of the two linear estimates.

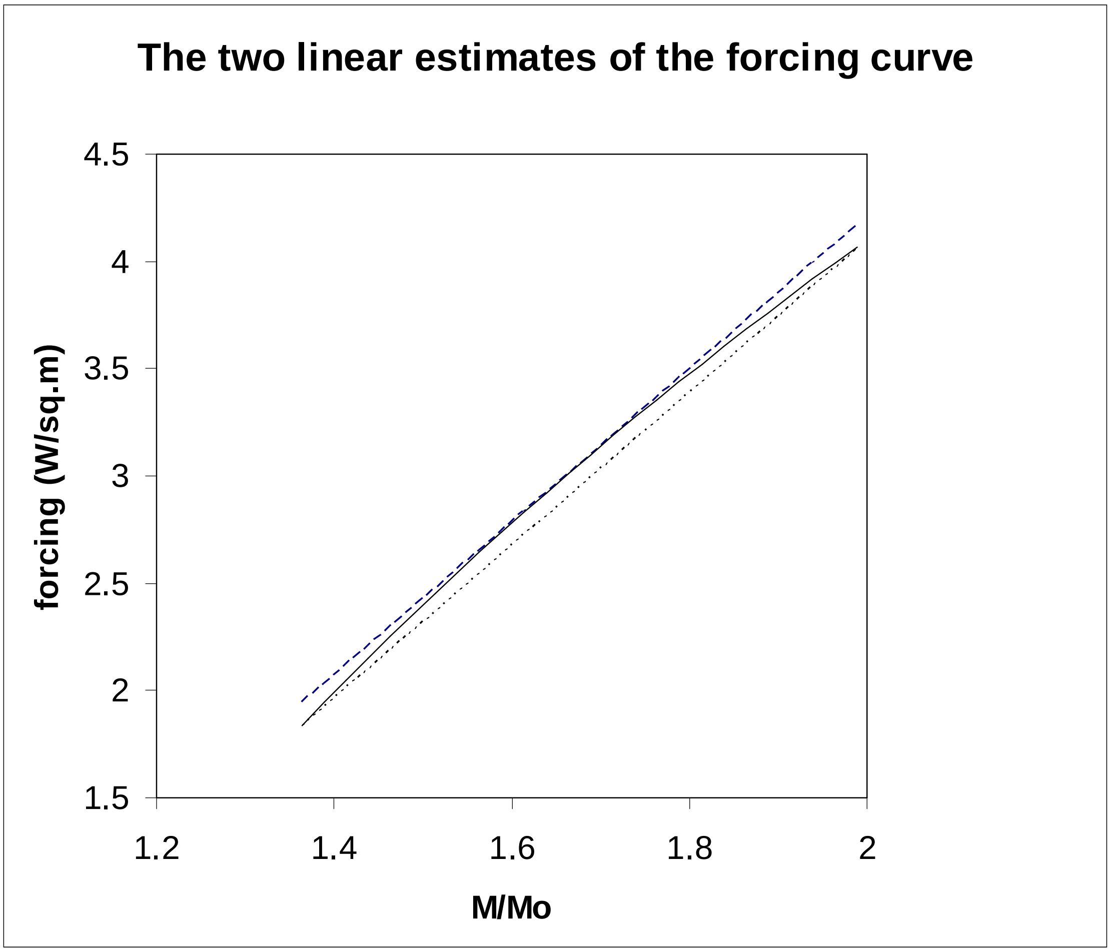

Figure 1. Illustration of the linearization of the radiative forcing function.

7.1.1.3 Temperature increase

In the TIMES Climate Module as in many other integrated models, climate change is represented by the global mean surface temperature. The idea behind the two-reservoir model is that a higher radiative forcing warms the atmospheric layer, which then quickly warms the upper ocean. In this model, the atmosphere and upper ocean form a single layer, which slowly warms the second layer consisting of the deep ocean.

$$\Delta T_{up}(y) = \Delta T_{up}(y-1) + \sigma_1 \{ F(y) - \lambda\, \Delta T_{up}(y-1) - \sigma_2\, [\Delta T_{up}(y-1) - \Delta T_{low}(y-1)] \} \tag{8}$$

$$\Delta T_{low}(y) = \Delta T_{low}(y-1) + \sigma_3\, [\Delta T_{up}(y-1) - \Delta T_{low}(y-1)] \tag{9}$$

with

- ΔT_{up} = globally averaged surface temperature increase above pre-industrial level,
- ΔT_{low} = deep-ocean temperature increase above pre-industrial level,
- σ_1 = 1-year speed of adjustment parameter for atmospheric temperature (also known as the *lag* parameter),
- σ_2 = coefficient of heat loss from atmosphere to deep oceans,
- σ_3 = 1-year coefficient of heat gain by deep oceans,
- λ = feedback parameter (climatic retroaction). It is customary to write λ as $\lambda = \gamma / C_s$, C_s being the *climate sensitivity* parameter, defined as the change in equilibrium atmospheric temperature induced by a doubling of CO_2 concentration.

Remark: in contrast with most other parameters, the value of C_s is highly uncertain, with a possible range of values from 1°C to 10°C. This parameter is therefore a prime candidate for sensitivity analysis, or for treatment by probabilistic methods.

The next five sections describe all the parameters, variables, and equations of the climate module. Section 7.2 presents the input parameters, section 7.3 the variables, section 7.4 the equations, and section 7.5 the reporting parameters. Section 7.6 shows the default values of the input parameters.

7.2 Parameters of the Climate Module

7.2.1 Overview

This and the next 4 sections describe the parameters, variables, and equations of the climate module. This section presents the input parameters, section 7.3 presents the variables, section 7.4 the equations, and section 7.5 the reporting parameters (i.e. expressions calculated in order to be reported as part of the solution, but not true GAMS equations). Section 7.6 shows the default values of the input parameters.

The distinction between variables and reporting parameters is important: while the former may be constrained or bounded, the latter are expressions that may contain true variables, but themselves may not be bounded. They may only be used for passive reporting purposes. In our implementation, only the concentrations of CO_2 in the three reservoirs are true variables, whereas radiative forcing and temperature changes are treated as reporting parameters[1].

General TIMES parameters (see chapter 3)

D(t): duration of period *t*, *t=1 to T*
B(t): first year in period *t*, *t=1 to T*
m(t): milestone year of period t (approximate middle year of period, defined as

$$m(t) = B(t) + \lfloor (D(t)-1)/2 \rfloor$$

y: designates a year, while *t* designates a period (ranging from 1 to T)
Y: designates the calibration year, which can be chosen by the user to be either *B*(1)–1, *m*(1)–1, or *m*(1), see section 7.7.3.

There are as many milestone years (middleyears) as there are periods (i.e. T). More precisely, there is a one-to-one mapping of milestoneyears (middleyears) to periods. Therefore, the period index t is sometimes replaced by the milestone year of that period m(t).

The input parameters are classified into in three categories (sections 7.2.2 to 7.2.4), for convenience and clarity. For completeness, section 7.2.6 gives the list of reporting parameters, which are described in detail in section 7.5

7.2.2 Deterministic input parameters

All input parameters are time-independent, except EXOFOR. Default values for all parameters are discussed in section 7.6. Note that, unless specified differently, all time dependent parameters represent values measured at the end of the year.

---

[1] Allowing bounds and constraints on these parameters would result in a non-linear, non-convex optimisation model

PHI_AT_UP, PHI_UP_AT, PHI_UP_LO, PHI_LO_UP (also denoted $\varphi_{atm\text{-}up}$, $\varphi_{up\text{-}atm}$, etc): annual CO2 flow coefficients between the three reservoirs (AT=Atmosphere, UP=Upper ocean layer, LO=Deep ocean layer). These are time-independent coefficients. Units: none

$CO2ATM^0$, $CO2UP^0$, $CO2LOW^0$: Values at the end of the calibration year Y (1995 by default) of the masses of CO_2 in the atmosphere, the upper ocean layer, and the deep ocean layer, respectively. Note that these values are time- indexed so that the model generator can pick up the correct value according to the calibration year (see subsection 7.7.1). Units: GtC

CO2_AT_PRE_IND: Pre-industrial atmospheric mass of CO2. Units = GtC

GAMMA (also denoted γ): radiative forcing sensitivity to a doubling of the atmospheric CO_2 mass. Units: Watts/m^2

LAMBDA (also denoted λ): a feedback parameter, representing the equilibrium impact of CO_2 concentrations doubling on climate. $\lambda = \gamma / C_s$, C_s being the temperature sensitivity to CO_2 concentrations doubling (^oC), and γ the radiative forcing sensitivity to CO_2 concentrations doubling (W/m^2). Note however that when C_s is assumed stochastic, its multiple values are specified via the generic S_CM_CONST parameter described below.

SIGMA1 (also denoted σ_1): speed of adjustment parameter for atmospheric temperature. $1/\sigma_1$ represents the thermal capacity of the atmospheric + upper ocean layer ($W\text{-}yr/m^2/^oC$). Note however that when SIGMA1 is assumed stochastic, its multiple values are specified via the generic S_CM_CONST parameter described below.

SIGMA2 (also denoted σ_2): ratio of the thermal capacity of the deep oceans to the transfer rate from shallow to deep ocean ($W/m^2/^oC$)

SIGMA3 (also denoted σ_3): $1/\sigma_3$ is the transfer rate (per year) from the upper level of the ocean to the deep ocean (yr^{-1})

DT_ATM^0, DT_LOW^0: values at the end of the calibration year Y (1995 by default) of the temperature changes (wrt to pre-industrial time) in atmosphere and deep layer, respectively. Units: ^oC

DT_FORC(t) is the total change in forcing in period t. Units: W/m2.

DT_TATM(t), and DT_TLOW(t): average global temperature changes in the atmosphere and in deep ocean respectively, in period t, relative to the average global temperatures in pre-industrial time. Units: ^oC

CM_EXOFOR(y): radiative forcing from Non-CO2 gases in each year from 1995. Units: Watts/m^2

CM_LINFOR(datayear,item,lim): lower and upper limit for the concentration of CO2 in atmosphere, used in the approximation of the radiative forcing equation. Item may be equal to CO2-ATM (in which case the limit is expressed as a ratio of concentration over pre-industrial concentration), or to CO2-PPM (in which case the limit is expressed in ppm of CO2-equivalent. The index lim is either equal to LO or to UP, depending on whther the lower or the upper limit of the range is being specified. For example, the following specifications may be used to selcet a range from 375 to 550 ppm for the approximation at year 2020.

CM_LINFOR(2020,CO2-PPM,LO)=375
CM_LINFOR(2020,CO2-PPM,UP)=550

Note that the values of LINFOR are systematically interpolated. The range can also be specified in a time-dependent manner taking into account the gradual increase in the expected range of possible concentration levels over time. That would further improve the accuracy of the linearization. For example, for 2005 the range could be specified to consist of only a single value, because the actual concentration in 2005 is well-known.

7.2.3 Upper bounds on climate variables

In TIMES, up to five climate upper bounds may be specified at any year. These upper bounds are specified via the single generic parameter CM_MAXC(datayear,item), where datayear is the year ar which the bound applies, and item may be any of the following five choices:

- **CO2-ATM:** for bounding the ratio of GHG concentration to the preindustrial concentration (denoted $\mathbf{M/M_0}$ in subsection 7.1.3)
- **CO2-PPM:** for bounding the GHG concentration expressed in ppm of CO2-equivalent
- **FORCING:** for bounding the atmospheric forcing expressed in W/ m^2
- **DELTA-ATM:** for bounding the change in global atmospheric temperature over pre-industrial temperature, expressed in ^{o}C. If this bound is used, the linearized forcing equation is used rather than the exact forcing equation.
- **CO2-GTC:** for bounding the global GHG emissions expressed in GtC.

The user may specify as many upper bounds as wanted.

7.2.4 Random climate parameters

If the stochastic programming version of TIMES is used, several climate parameters may be assumed random. These fall into two categories: the upper bounds on climate quantities discussed in section 7.2.2, and two climate coefficients, **Cs** and **SIGMA1.**

Regarding the random upper bounds, their multiple values are specified via the stochastic version of the **CM_MAX** parameter, namely **S_CM_MAX(datayear,item,stage,sow)**, where in addition to **datayear** and **item** already explained, **stage** refers to the stage of the event tree and **sow** refers to the state-of-the-world. Note that this single generic parameter will be specified as many times as there are **stages** and **sow**'s in the stochastic event tree. If this parameter is specified, the corresponding values of the deterministic parameter **CM_MAX** are superseded.

Regarding the two random coefficients, their multiple values are then declared via the single generic parameter **S_CM_CONST(item,stage,sow)**, where **item** may be equal to **CS** or to **SIGMA1**, **stage** is the stage number, and **sow** is the state-of-the-world. Note that this single generic parameter will be specified as many times as there are stages and sow's in the stochastic event tree. If this parameter is specified, the corresponding values of the deterministic parameter (**LAMBDA** and/or **SIGMA1**) are superseded.

The reader is referred to the documentation of the stochastic programming version of TIMES for the precise meaning of the **stage** and **sow** concepts.

Remark: in addition to the possible values of the random parameters, the user must specify the probabilities attached to each ***sow***. This is also explained in the chapter on stochastic TIMES.

7.2.5 Internal parameters

In order to facilitate the writing of the equations, several intermediate quantities are constructed. These are described in each equation section. An additional internal parameter CM_DEFAULT is used to store the default values for the calibration quantities (see section 7)

7.2.6 Reporting parameters

There are two reporting parameters, CM_RESULT and CM_MAXC_M, which contain the results on the levels of the climate variables (or reporting quantities) and the dual values of the constraints defined by using CM_MAXC.

CM_RESULT is indexed by year y and result type {CO2ATM, DELTA_FORC, DELTA_ATM, DELTA_LO}, the expressions for which are given in section 7.5. The values represent the quantities at the end of year y.

- CO2ATM(y): the value of the atmospheric mass of CO2-equivalent at the end of year **y**, obtained by interpolating the values of the variable **VAR_CO2UP(t)** at the two relevant milestone years.
- DELTA_FORC(y): exact forcing value at end of year y, calculated using the logaritm forcing equation defined in section 7.3 and the CO2ATM(y) value.
- DELTA_ATM(y): exact atmospheric temperature value at end of year y, calculated using the forcing DT_FORC(y).
- DELTA_LOW(y): exact lower ocean temperatura value at end of year y, calculated using the forcing DT_FORC(y).

CM_MAXC_M is indexed by year y and constraint type {CO2-GTC, CO2-ATM, CO2-PPM, DELTA_ATM, FORCING}. The values represent directly the dual values of these constraints at year y.

7.3 Climate related Variables

There are two families of Climate variables, both indexed by ***item*** (the associated climate quantity) and ***t*** (time period).

7.3.1 VAR_CLITOT(item, t)

Description: Global average annual level of the climate-related quantity item during period t. Used for the following predefined item indexes:

- CO2-GTC: Atmospheric CO2 emissions at period t. Units: GtC.
- CH4-MT: Atmospheric CH4 emissions at period t. Units: Mt.
- N2O-MT: Atmospheric N2O emissions at period t. Units: Mt.
- FORCING: Forcing at period t, as calculated by the linear approximation of section 7.1.3. Units: W/m^2

Notes: The name *VAR_CO2TOT* is below used as an alias for *VAR_CLITOT(CO2-GTC)*. The indexes CH4-MT and N2O-MT are used only if the user has explicitly indicated that the concentrations of CH_4 and N_2O are to be modeled separately. By default these emissions are assumed to be included as CO_2 equivalents in the variables for CO2-GTC.

7.3.2 VAR_CLIBOX(item, t)

Description: These variables represent the levels of concentration and temperature change in the reservoir models for concentration/temperature. For concentrations, the variables represent the average masses of CO_2 in period t in the three quickly mixing reservoirs respectively corresponding to the atmosphere, the upper level of the ocean, and the deep oceanic layer. Whenever necessary, the average mass in period t is also interpreted as the mass at the end of the middle year y of period t. Units: GtC. For temperature, the variables represent the changes in atmospheric temperature and in lower ocean temperature at period t, over the pre-industrial values, as calculated using the linear approximation of forcing. Units: ^oC. The following predefined item indexes are used:

- CO2-ATM: Atmospheric CO_2 concentration at period t. Units: GtC.
- CO2-UP: Upper oceanic layer CO_2 concentration at period t. Units: GtC.
- CO2-LO: Deep oceanic layer CO_2 concentration at period t. Units: GtC.
- DELTA-ATM: Change in atmospheric temperature in period t. Units: ^oC.
- DELTA-LO: Change in lower ocean temperature in period t. Units: ^oC.

In subsequent sections, the following shorthand notation is used for convenience:

Shorthand name	**Name used in model generator**
VAR_CO2ATM_t	*VAR_CLIBOX(CO2-ATM, t)*
VAR_CO2UP_t	*VAR_CLIBOX(CO2-UP, t)*
VAR_CO2LOW_t	*VAR_CLIBOX(CO2-LO, t)*

7.4 Climate Equations

There are seven definitional equations: the first equation calculates the global emissions of GHG (in CO_2-eq), the next three equations calculate the concentrations of CO2-eq in the three reservoirs, the fifth calculates the linearized forcing at period t. The sixth calculates the atmospheric temperature at period t, and the seventh calculates the lower ocean temperature at period t.

In addition, there are up to five generic equations expressing the upper bounding of the five climate quantities discussed in subsection 7.2.3. This generic equation is generated as many times as an upper bound on any climate variable is specified by the user, and is not generated if no upper bound is specified.

We now give the precise formulations of these constraints.

7.4.1 Equation: EQ_CLITOT(CO2-GTC)

Indices: period (t)

Type: =

Related variables: VAR_COMNET(r,t,c,s)

Units: Gt carbon

Purpose: This equation defines the global CO2 emissions into the atmosphere during each period t (expressed in GtC), as a function of the atmospheric CO2 emissions in each region (which are represented by existing TIMES variables VAR_COMNET).

Remark:

§ A user provided conversion factor (CO2GTC(r,c)) is required to convert emission units used in the TIMES model to Gigatonnes of carbon. For instance, if CO2 emissions are in Mt CO2, the conversion coefficient must be taken equal to: 0.00027272727. CO2GTC also identifies the emission commodities in the model that should be converted into carbon emissions.

Equation

$$EQ\_CLITOT(CO2\text{-}GTC)_t \quad \ni \quad (t \in MILESTONEYEARS)$$

$$-VAR\_CO2TOT(t) + \sum_{r \in R, c \in COM} CO2GTC(r,c) \bullet \sum_{s \in S} VAR\_COMNET(r,t,c,s) = 0$$

where R is the set of regions, and S is the set of timeslices

7.4.2 Equation: EQ_CLICONC(ATM)

Indices: **period (t)**

Type: =

Related variables: VAR_CO2ATM, VAR_CO2UP , VAR_CO2LOW, VAR_CO2TOT

Purpose: This equation defines the average mass of CO2 in the atmosphere during period *t*, which may also be interpreted (by convention) as the atmospheric CO2 mass at the end of middleyear *m(t)*. This mass is expressed as a function of the average masses of CO2 in the three reservoirs during period *t–1* and of the average CO2 emissions between the previous middleyear *m(t–1)* and the current middleyear *m(t)*.

Remarks:

- The coefficients governing this equation depend on the lengths of periods *t* and *t–1,* and therefore require the computation of intermediate quantities. We follow the TIMES convention that each period *t* is represented by its milestone year *m(t)*, situated at or near the middle of the period. This explains the fact that the concentration at period *t* depends on emissions at periods *t–1* and *t*.
- The corresponding equations of the RICE-99 model are simpler for two reasons: first, RICE has constant period length, and second, in RICE, the concentration variables represent the first year of the period, rather than at the end of the middle year as in TIMES.

Equation:

$$EQ\_CLICONC(ATM)_t \quad \ni (t \in MILESTONEYEARS) \wedge (t \neq 1):$$

$$\begin{aligned}
&- VAR\_CO2ATM(t) \\
&+ A_{11}(t) \times VAR\_CO2ATM(t-1) \\
&+ A_{12}(t) \times VAR\_CO2UP(t-1) \\
&+ A_{13}(t) \times VAR\_CO2LOW(t-1) \\
&+ BB_{11}(t) \times VAR\_CO2TOT(t) \\
&+ CC_{11}(t) \times VAR\_CO2TOT(t-1) \\
&= 0
\end{aligned}$$

$$EQ\_CO2ATM_t \quad \ni \ (t = 1):$$

$$\begin{aligned}
&VAR\_CO2ATM(t) = BB_{11}(t) \times VAR\_CO2TOT(t) + \\
&A_{11}(t) \times CO2ATM_{\mathrm{Y}}^{0} + A_{12}(t) \times CO2UP_{\mathrm{Y}}^{0} + A_{13}(t) \times CO2LOW_{\mathrm{Y}}^{0}
\end{aligned}$$

where:

$\{A_{11}(t), A_{12}(t), A_{13}(t)\}$ is the first row of matrix: $PHI^{n(t)} \quad (PHI^{0} = I)$

PHI is the 3×3 matrix: $\begin{bmatrix} (1-PHI\_AT\_UP) & PHI\_UP\_AT & 0 \\ PHI\_AT\_UP & (1-PHI\_UP\_AT-PHI\_UP\_LO) & PHI\_LO\_UP \\ 0 & PHI\_UP\_LO & (1-PHI\_LO\_UP) \end{bmatrix}$

$BB_{11}(t)$ is the first coefficient of the first column of matrix:

$$BB(t) = \sum_{i=0}^{p(t)-1} PHI^{i} \qquad if \ \ p(t) \geq 1$$

$$BB(t) = 0 \qquad if \ \ p(t) = 0$$

$CC_{11}(t)$ is the first coefficient of the first column of matrix:

$$CC(t) = \left(PHI^{p(t)} + \ldots + PHI^{n(t)-1}\right) \qquad if \ \ n(t) \geq p(t) + 1$$

$$CC(t) = 0 \qquad if \ \ n(t) = p(t)$$

Note that $p(t) = 1$ *occurs when period* t *has a length of* 1 *year*

Note that $n(t) = p(t)$ *occurs when period* $t-1$ *has a length of* 1 *year*

$$p(t) = \left\lfloor \frac{D(t)+1}{2} \right\rfloor, \qquad n(t) = m(t) - m(t-1) \qquad if \ t \neq 1,$$

$$p(t) = m(t) - \mathrm{Y}, \qquad n(t) = p(t) \qquad if \ t = 1$$

$D(t)$ is the number of years in period t, *and* $m(t)$ is the middle year of period t defined as

$$m(t) = B(t) + \left\lfloor \frac{D(t)-1}{2} \right\rfloor$$

$\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x (*the FLOOR function*)

7.4.3 Equation: EQ_CLICONC(UP)

Indices: period (t)

Type: =

Related variables: VAR_CO2ATM(t), VAR_CO2UP(t), VAR_CO2LOW(t), VAR_CO2TOT(t)

Purpose: This equation defines the average mass of CO2 in the upper ocean layer during period *t*, which may also be interpreted (by convention) as the CO2 mass in the upper ocean layer at the end of middleyear *m(t)*. This mass is expressed as a function of the average masses of CO2 in the three reservoirs during period *t–1* and of the average CO2 emissions between the previous middleyear *m(t–1)* and the current middleyear *m(t)*.

Remarks:

- The coefficients governing this equation depend on the lengths of periods *t* and *t–1*, and therefore require the computation of intermediate quantities. We follow the TIMES convention that each period *t* is represented by its milestone year *m(t)*, situated at or near the middle of the period. This explains the fact that the concentration at period *t* depends on emissions at periods *t–1 and t*.
- The corresponding equations of the RICE-99 model are simpler for two reasons: first, RICE has constant period length, and second, in RICE, the concentration variables represent the first year of the period, rather than the middle year as in TIMES.

Equation:

$EQ\_CLICONC(UP)_t \quad \ni (t \in MODELYEARS) \wedge (t \neq 1):$

$$
\begin{aligned}
&- VAR\_CO2UP(t) \\
&+ A_{21}(t) \times VAR\_CO2ATM(t-1) \\
&+ A_{22}(t) \times VAR\_CO2UP(t-1) \\
&+ A_{23}(t) \times VAR\_CO2LOW(t-1) \\
&+ BB_{21}(t) \times VAR\_CO2TOT(t) \\
&+ CC_{21}(t) \times VAR\_CO2TOT(t-1) \\
&= 0
\end{aligned}
$$

$EQ\_CO2UP_t \quad \ni (t = 1):$

$$
\begin{aligned}
&VAR\_CO2UP(t) = BB_{21}(t) \times VAR\_CO2TOT(t) + \\
&\quad A_{21}(t) \times CO2ATM^0_{\mathrm{Y}} + A_{22}(t) \times CO2UP^0_{\mathrm{Y}} + A_{23}(t) \times CO2LOW^0_{\mathrm{Y}}
\end{aligned}
$$

where:

$\{A_{21}(t), A_{22}(t), A_{23}(t)\}$ is the second row of matrix: $PHI^{n(t)} \; (PHI^0 = I)$

PHI is the 3×3 matrix: $\begin{bmatrix} (1-PHI\_AT\_UP) & PHI\_UP\_AT & 0 \\ PHI\_AT\_UP & (1-PHI\_UP\_AT-PHI\_UP\_LO) & PHI\_LO\_UP \\ 0 & PHI\_UP\_LO & (1-PHI\_LO\_UP) \end{bmatrix}$

$BB_{21}(t)$ is the second coefficient of the first column of matrix:

$$
\begin{aligned}
&BB(t) = \sum_{i=0}^{p(t)-1} PHI^i && if \;\; p(t) \geq 1 \\
&BB(t) = 0 && if \;\; p(t) = 0
\end{aligned}
$$

$CC_{21}(t)$ is the second coefficient of the first column of matrix:

$$
\begin{aligned}
&CC(t) = \left(PHI^{p(t)} + \cdots + PHI^{n(t)-1}\right) && if \;\; n(t) \geq p(t) + 1 \\
&CC(t) = 0 && if \;\; n(t) = p(t)
\end{aligned}
$$

$$p(t) = \left\lfloor \frac{D(t)+1}{2} \right\rfloor, \qquad n(t) = m(t) - m(t-1) \qquad if \; t \neq 1,$$

$$p(t) = m(t) - \mathrm{Y}, \qquad n(t) = p(t) \qquad if \; t = 1$$

$D(t)$ is the number of years in period t, *and* $m(t)$ is the middle year of period t defined as

$$m(t) = B(t) + \left\lfloor \frac{D(t)-1}{2} \right\rfloor$$

$\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x

7.4.4 Equation: EQ_CLICONC(LO)

Indices: period (t)

Type: =

Related variables: VAR_CO2ATM(t), VAR_CO2UP(t), VAR_CO2LOW(t), VAR_CO2TOT (t)

Purpose: This equation defines the average mass of CO2 in the lower ocean layer during period *t*, which may also be interpreted (by convention) as the CO2 mass in the lower ocean layer at the end of middleyear *m(t)*. This mass is expressed as a function of the average masses of CO2 in the three reservoirs during period *t–1* and of the average CO2 emissions between the previous middleyear *m(t–1)* and the current middleyear *m(t)*.

Remarks:

- The coefficients governing this equation depend on the lengths of periods *t* and *t–1,* and therefore require the computation of intermediate quantities. We follow the TIMES convention that each period *t* is represented by its milestone year *m(t)*, situated at or near the middle of the period. This explains the fact that the concentration at period *t* depends on emissions at periods *t–1 and t*.
- The corresponding equations of the RICE-99 model are simpler for two reasons: first, RICE has constant period length, and second, in RICE, the concentration variables represent the first year of the period, rather than the middle year as in TIMES.

Equation:

$EQ\_CLICONC(LO)_t \quad \ni (t \in MODELYEARS) \wedge (t \neq 1):$

$$
\begin{aligned}
& - VAR\_CO2LOW(t) \\
& + A_{31}(t) \times VAR\_CO2ATM(t-1) \\
& + A_{32}(t) \times VAR\_CO2UP(t-1) \\
& + A_{33}(t) \times VAR\_CO2LOW(t-1) \\
& + BB_{31}(t) \times VAR\_CO2TOT(t) \\
& + CC_{31}(t) \times VAR\_CO2TOT(t-1) \\
& = 0
\end{aligned}
$$

$EQ\_CO2LOW_t \quad \ni (t = 1):$

$$
\begin{aligned}
& VAR\_CO2LOW(t) = BB_{31}(t) \times VAR\_CO2TOT(t) + \\
& \quad A_{31}(t) \times CO2ATM^{0}_{\Upsilon} + A_{32}(t) \times CO2UP^{0}_{\Upsilon} + A_{33}(t) \times CO2LOW^{0}_{\Upsilon}
\end{aligned}
$$

where:

$\{A_{31}(t), A_{32}(t), A_{33}(t)\}$ is the third row of matrix: $PHI^{n(t)} \quad (PHI^{0} = I)$

PHI is the 3×3 matrix: $\begin{bmatrix} (1-PHI\_AT\_UP) & PHI\_UP\_AT & 0 \\ PHI\_AT\_UP & (1-PHI\_UP\_AT-PHI\_UP\_LO) & PHI\_LO\_UP \\ 0 & PHI\_UP\_LO & (1-PHI\_LO\_UP) \end{bmatrix}$

$BB_{31}(t)$ is the third coefficient of the first column of matrix:

$$
\begin{aligned}
& BB(t) = \sum_{i=0}^{p(t)-1} PHI^{i} && \text{if } p(t) \geq 1 \\
& BB(t) = 0 && \text{if } p(t) = 0
\end{aligned}
$$

$CC_{31}(t)$ is the third coefficient of the first column of matrix:

$$
\begin{aligned}
& CC(t) = \left(PHI^{p(t)} + \cdots + PHI^{n(t)-1}\right) && \text{if } n(t) \geq p(t) + 1 \\
& CC(t) = 0 && \text{if } n(t) = p(t)
\end{aligned}
$$

$p(t) = \left\lfloor \dfrac{D(t)+1}{2} \right\rfloor, \qquad n(t) = m(t) - m(t-1) \qquad \text{if } t \neq 1,$

$p(t) = m(t) - \Upsilon, \qquad n(t) = p(t) \qquad \text{if } t = 1$

$D(t)$ is the number of years in period t, *and* $m(t)$ is the middle year of period t defined as

$m(t) = B(t) + \left\lfloor \dfrac{D(t)-1}{2} \right\rfloor$

$\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x

Remark: The three concentration equations may be written as a single vector equation, as follows, using the notation described above:

$$-\begin{bmatrix} \boldsymbol{VAR\_CO}21\boldsymbol{ATM}(\boldsymbol{t}) \\ \boldsymbol{VAR\_CO}2\boldsymbol{UP}(\boldsymbol{t}) \\ \boldsymbol{VAR\_CO}2\boldsymbol{LOW}(\boldsymbol{t}) \end{bmatrix} + \boldsymbol{PHI}^{\boldsymbol{n}} \bullet \begin{bmatrix} \boldsymbol{VAR\_CO}21\boldsymbol{ATM}(\boldsymbol{t}-1) \\ \boldsymbol{VAR\_CO}2\boldsymbol{UP}(\boldsymbol{t}-1) \\ \boldsymbol{VAR\_CO}2\boldsymbol{LOW}(\boldsymbol{t}-1) \end{bmatrix} + \boldsymbol{BB} \bullet \begin{bmatrix} \boldsymbol{VAR\_CO}2\boldsymbol{TOT}(\boldsymbol{t}) \\ 0 \\ 0 \end{bmatrix} + \boldsymbol{CC} \bullet \begin{bmatrix} \boldsymbol{VAR\_CO}2\boldsymbol{TOT}(\boldsymbol{t}-1) \\ 0 \\ 0 \end{bmatrix} =$$

It is interesting to compare this to the RICE-99 vector equation (Nordhaus and Boyer, 1999), which writes:

$$-\begin{bmatrix} \boldsymbol{VAR\_CO}21\boldsymbol{ATM}(\boldsymbol{t}) \\ \boldsymbol{VAR\_CO}2\boldsymbol{UP}(\boldsymbol{t}) \\ \boldsymbol{VAR\_CO}2\boldsymbol{LOW}(\boldsymbol{t}) \end{bmatrix} + \boldsymbol{PHI}10 \bullet \begin{bmatrix} \boldsymbol{VAR\_CO}21\boldsymbol{ATM}(\boldsymbol{t}-1) \\ \boldsymbol{VAR\_CO}2\boldsymbol{UP}(\boldsymbol{t}-1) \\ \boldsymbol{VAR\_CO}2\boldsymbol{LOW}(\boldsymbol{t}-1) \end{bmatrix} + 10 \cdot \begin{bmatrix} \boldsymbol{VAR\_CO}2\boldsymbol{TOT}(\boldsymbol{t}-1) \\ 0 \\ 0 \end{bmatrix} = 0$$

where ***PHI*10** is the 10-year transition matrix adopted in RICE-99. As explained before, the simpler RICE equation is due to the constant period length (10 years) and the fact that the milestone year of RICE represents the first year of a period.

7.4.5 EQ_CLITOT(FORCING)

Indices: period (t)

Type: =

Related variables: VAR_CO2ATM(t)

Purpose: expresses the value of the atmospheric forcing at period **t**, as evaluated by the linear approximation discussed in section 7.3.

Equation:

$$EQ\_CLITOT(FORCING)_t \qquad \ni \quad 1 \le t \le T$$

$$VAR\_CLITOT(FORCING)_t = \frac{F1 + F2}{2}$$

$$F1 = \frac{GAMMA}{\ln 2} \times \left[\ln \frac{GAMMA}{slope \times \ln 2 \times CO2\_PREIND} - 1 \right] + VAR\_CO2ATM_t \times slope$$

$$F2 = \frac{GAMMA}{\ln 2} \times \ln \frac{CONV \times CM\_LINFOR(m(t), item, LO)}{CO2\_PREIND}$$

$$+\, slope \times \left[VAR\_CO2ATM_t - CONV \times CM\_LINFOR(m(t), item, LO) \right]$$

$where:$

$$slope = \frac{GAMMA}{\ln 2} \times \frac{\ln \dfrac{CM\_LINFOR(m(t), item, UP)}{CM\_LINFOR(m(t), item, LO)}}{CONV \times \left[CM\_LINFOR(m(t), item, UP) - CM\_LINFOR(m(t), item, LO) \right]}$$

$and:$

$$\begin{aligned} CONV &= 1 && \text{if} \quad item = CO2\text{-}ATM \\ &= 2.13 && \text{if} \quad item = CO2\text{-}PPM \end{aligned}$$

Remark

§ *slope* is the slope of the chord drawn on the graph of the exact forcing curve, from the lower to the upper limits of the range

§ *CONV* simply converts the bounds into GtC

7.4.6 EQ_CLITEMP(ATM)

Indices: **period (t)**

Sign: =

Purpose: This equation defines the temperature change of the atmospheric layer (including upper ocean) at period **t** over the pre-industrial temperature. It is expressed as the value of the temperature at the end of year m(t), as a function of the temperature changes in the two layers at the previous period, and of the change in radiative forcing at the current period, as expressed by the linearized expression in section 7.5.

Equation:

$EQ\_CLITEMP(ATM)_t \quad \ni 1 \le t \le T$

$VAR\_CLIBOX(DELTA\text{-}ATM)_t = DT\_ATM_{m(t)}$

where

$m(t)$ *is the milestone year of period* t

and

$DT\_ATM_y \quad \ni (B(1) \le y \le EOH)$ *is defined as*

$$\begin{aligned} DT\_ATM_y \; &= (1 - LAMBDA \times SIGMA1 - SIGMA1 \times SIGMA2) \times DT\_ATM_{y-1} \\ &+ SIGMA1 \times SIGMA2 \times DT\_LOW_{y-1} \; + \; SIGMA1 \times FORCING_y \end{aligned}$$

$$DT\_ATM_{B(1)-1} = DT\_ATM\_0$$

where

$FORCING_y =$ interpolation of the values of the $VAR\_CLITOT(FORCING)_t$ variables

7.4.7 EQ_CLITEMP(LO)

Indices: **period (t)**

Sign: =

Purpose: This parameter defines the temperature change of the lower ocean layer over its pre-industrial temperature measured at the end of milestoneyear m(t), as a function of the temperature changes in the two layers at the end of the previous year.

Equation:

$$EQ\_CLITEMP(LO)_t \quad \ni \; 1 \le t \le T$$

$$VAR\_CLIBOX\,(DELTA\text{-}LO)_t = DT\_LOW_{m(t)}$$

$$and \;\; DT\_LOW_y \quad \ni (B(1) \le y \le EOH)$$

$$DT\_LOW_y \;\; = SIGMA3 \times DT\_ATM_{\,y-1} + (1 - SIGMA3) \times DT\_LOW_{y-1}$$

$$and$$

$$DT\_LOW_{B(1)-1} \; = \;\; DT\_LOW\_0$$

7.4.8 EQ_CLIMAX

Indices: Climate variable (item), datayear (y)

Sign: ≤

Purpose: This generic bounding equation can be used to impose an upper bound on any or all of the climate variables: *VAR_CLITOT(CO2-GTC)*, *VAR_CLITOT(FORCING)*, *VAR_CLIBOX(CO2-ATM)* or *VAR_CLIBOX(DELTA-ATM)*, at any year of the model's horizon until *m(T)*. Any *CM_MAXC* parameters specified with the item *CO2-PPM* are converted to equivalent attributes specified by using *CO2-ATM*.

Equation

$$EQ\_CLIMAX(item)_y \qquad \ni \; m(1) \le y \le m(T)$$

$$\alpha_y VAR\_XXX_{t-1} + \beta_y VAR\_XXX_t \le CM\_MAXC(y, item)$$

$$\begin{array}{lll} where & & \\ & m(t-1) < y \le m(t), & \\ & \alpha_y (m(t) - y) + \beta_y (y - m(t-1)) = y, & \\ XXX & = CLITOT(CO2\text{-}GTC) & if \;\; item = CO2\text{-}GTC \\ & = CLITOT(FORCING) & if \;\; item = FORCING \\ & = CLIBOX(CO2\text{-}ATM) & if \;\; item = CO2\text{-}ATM \\ & = CLIBOX(DELTA\text{-}ATM) & if \;\; item = DELTA\text{-}ATM \end{array}$$

Remark: if stochastic TIMES is activated and some climate upper bounds are assumed random, the expressions above are altered to reflect the different values of the upper bounds at some stages and for some sow's. Please refer to the documentation on stochastic TIMES, Chapter ?? of this manual.

7.5 Reporting parameters

7.5.1 CO2ATM

Indices: **modelyear(y)**

Purpose: this parameter is defined as the CO2 mass in atmosphere at the end of year y. It is equal to the linearly interpolated value of VAR_CO2ATM(t) and VAR_CO2ATM(t-1), where t is such that **y** lies in the interval [m(t-1)+1, m(t)]. The parameter is used for calculating the parameter DT_FORC(y) described in subsection 7.5.2.

Expression

$$CO2ATM_y \ni \left(\mathrm{Y} \le y \le m(T)\right)$$

$$if \;\; m(t-1) \le y \le m(t) \;\; for\,some \;\; t \ge 2, \; then:$$

$$CO2ATM_y \; = \; VAR\_CO2ATM_t \times \frac{y - m(t-1)}{m(t) - m(t-1)} \; + \; VAR\_CO2ATM_{t-1} \times \frac{m(t) - y}{m(t) - m(t-1)}$$

$$if \;\; \mathrm{Y} \le y < m(1) \;\;\; then:$$

$$CO2ATM_y \; = \; VAR\_CO2ATM_1 \times \frac{y - \mathrm{Y}}{m(1) - \mathrm{Y}} \; + CO2ATM_{\mathrm{Y}}^{0} \times \frac{m(1) - y}{m(1) - \mathrm{Y}}$$

7.5.2 DT_FORC

Indices: **modelyear(y)**

Purpose: This reporting parameter defines the total increase in radiative forcing due to anthropogenic gases in the atmosphere, measured at the end of year **y**, relative to pre-industrial times, using the exact logarithmic function discussed in section 7.1.3. It decomposes into two main terms: forcing due to atmospheric CO_2 concentration (which is endogenous in TIMES), and forcing due to other sources (which is considered exogenous in TIMES, unless other gases have been represented as their CO2-equivalent). The parameter is calculated at the end of every year within the horizon.

Expression

$DT\_FORC_y \ni (\mathrm{Y} \leq y \leq m(T))$

$$DT\_FORC_y = \frac{GAMMA}{\ln 2} \bullet \ln \frac{CO2ATM_y}{CO2\_PREIND} + EXOFORC_y$$

where :

$GAMMA$ is the forcing sensitivity to a doubling in atmospheric CO2 concentration

$CO2\_PREIND$ is the mass of atmospheric CO2 at pre - industrial times

$EXOFORC_y$ is the contribution of sources other than CO2 to the change in forcing at the end of year y

7.5.3 DT_ATM

Indices: modelyear(y)

Purpose: This reporting parameter defines the temperature change of the atmospheric layer (including upper ocean) over the pre-industrial temperature, measured at the end of each year y. It is a function of the temperature changes in the two layers at the end of the previous year, and of the change in radiative forcing at the end of the previous year.

Expression

$DT\_ATM_y \ni (\mathrm{Y} < y \leq m(T))$:

$$\begin{aligned} DT\_ATM_y = {} & (1 - LAMBDA \times SIGMA1 - SIGMA1 \times SIGMA2) \times DT\_ATM_{y-1} \\ & + SIGMA1 \times SIGMA2 \times DT\_LOW_{y-1} + SIGMA1 \times DT\_FORC_y \end{aligned}$$

and

$DT\_ATM_y \ni (y = \mathrm{Y})$:

$$DT\_ATM_{\mathrm{Y}} = DT\_ATM^0_{\mathrm{Y}}$$

where :

$1/SIGMA1$ is the one - year thermal capacity coefficient of the atmospheric layer

$1/SIGMA2$ is the one - year temperature transfer rate from atmospheric layer to lower ocean layer

$LAMBDA$ is a parameter representing the equilibrium impact of atmospheric CO2 concentration doubling on global atmospheric temperature. Note that if Cs represents the sensitivity of global temperature to a doubling in CO2 concentration, the following relationship holds : $LAMBDA = GAMMA / Cs$

7.5.4 DT_LOW

Indices: modelyear(y)

Purpose: This parameter defines the temperature change of the lower ocean layer over its pre-industrial temperature, measured at the end of year y. It is a function of the temperature changes in the two layers at the end of the previous year.

Expression

$DT\_LOW_y \ni (\mathrm{Y} < y \le m(T))$:

$$DT\_LOW_y = SIGMA3 \times DT\_ATM_{y-1} + (1 - SIGMA3) \times DT\_LOW_{y-1}$$

and

$DT\_LOW_y \ni (y = \mathrm{Y})$:

$$DT\_LOW_{\mathrm{Y}} = DT\_LOW^{0}_{\mathrm{Y}}$$

where :

*SIGMA*3 is the (dimensionless) ratio of the thermal capacity of the lower ocean layer to the transfer rate from atmospheric to lower ocean layer.

7.6 User constraint parameter

7.6.1 UC_CLI

Indices: uc_n, side, reg, y, item

Purpose: This parameter can be used to define climate variable coefficients in any user constraints. The ***item*** index can be any of the following climate variables:

- CO2-GTC - total global CO2 emissions (or CO2-eq. GHGs)
- CO2-ATM - CO2 concentration in the atmosphere
- CO2-UP - CO2 concentration in the biosphere/upper ocean
- CO2-LO - CO2 concentration in the deep ocean layer
- FORCING - radiative forcing
- DELTA-ATM - atmospheric temperature
- DELTA-LO - deep oceanic temperature

As the attribute has a region index, it can be used for defining custom relationships by each region, between any of the climate variables and e.g. some process flows, activities or capacities, or total commodity flows. However, if used in a global constraint, one should normally specify the UC_CLI attribute only for one region (e.g. GLB).

7.7 Default values of the climate parameters

Table 7.1 shows the assumed values of all parameters of the Climate Module except exogenous forcing.

Table 7.1. Parameters of the climatic module (default values)
(Notation is that of the GAMS program)

Parameter	**Default value**
Gamma	3.71 W/m^2
PHI_UP_AT	0.0453 per year
PHI_AT_UP	0.0495 per year
PHI_LO_UP	0.00053 per year
PHI_UP_LO	0.0146 per year
C_s not directly needed	2.91 °C
LAMBDA	1.41
SIGMA1	0.024 per year
SIGMA2	0.44 (no time dimension)
SIGMA3	0.002 per year
CO2ATM_PRE_IND	596.4 GtC (pre-industrial equilibrium)
CO2_ATM_0	742 GtC (in 1995)
CO2_UP_0	781 GtC (in 1995)
CO2_LO_0	19230 GtC (in 1995)
DELTAT_ATM_0	0.43 °C (1995)
DELTAT_LOW_0	0.06 °C (1995)
DELTAFORCING_0 not directly needed	1.0395 (1995)

Notes :

1. The last 6 parameters are given for year 1995. Therefore 1995 is considered to be the last year before the first period (in other words B(1)-1). Users whose first period does not start in 1996 must provide appropriate values.
2. Cs and SIGMA1 may be assumed random, in which case the default values are not used. The user must specify their values explicitly.
3. EXOFORCING: Nordhaus and Boyer use the following formula to calculate the radiative forcing due to all GHG's except CO_2. In the case of the TIMES global model, the energy related methane and N_2O emissions are already accounted for in the calculation of CO_2 equivalent emissions. Therefore, the formula below constitutes an upper bound for the radiative forcing due to other gases.

$$\mathrm{EXOFORCING}(y) = \begin{cases} -0.1965 + 0.013465 * (y-1995), & \text{if } 1995 \le y \le 2095 \\ 1.15, & \text{if } y > 2095 \end{cases}$$

In Nordhaus and Boyer (1999), the forcings of other GHGs (CFCs, CH4, N2O, ozone), and from aerosols are considered to be exogenous. Some of these gases are poorly understood. Moreover, some of them are controlled by non-climate policies (e.g. CFC's, ozone, aerosols). These values are inspired by the MAGICC model (Wigley *et al.*, 1994). The IPCC TAR (2001) provides estimated ranges of the forcing of non-CO_2 GHGs in 1998 (chapter 6) as well as simplified equations (Table 6.2 of chapter 6). However, no average value is provided for several of the gases, which makes it difficult to compare the exogenous forcing proposed by Nordhaus and Boyer (1999) to these updated estimations.

In recent (2006) applications of the TIMES Integrated Assessment Model (TIAM), the GHG's considered included all anthropogenic emissions of CO_2, CH_4, and N_2O, converted into CO2 equivalent. Therefore, the exogenous forcing y=used in these runs was quite different from that of Nordhaus and Boyer. A constant value of 0.4 W/m2 was assumed over the entire horizon (2000-2100) and beyond.

7.8 GAMS implementation

All required GAMS modules have been added to the code. The Climate Module is implemented as a TIMES extension module, which is a modular way of adding code to the standard TIMES code. The Climate Module is activated by adding the 'short name' of the climate extension (namely: **CLI)** to the arguments of the call for **initmty.mod**. Thus, the Climate Module is activated by the following call statement in the run file:

```
$ BATINCLUDE initmty.mod CLI
```

All the required climate parameters must also be specified, as explained in section 2.1, and the climate results are made available to the VEDA-BE report generator as explained in section 2.4.

7.8.1 Specification of parameters

User input parameters

All parameters are also available from the VEDA-FE shell, where they may be modified. The Appendix to section 2 shows the excel template used by VEDA to initially import the parameter values. Thereafter, these values may be modified via the VEDA-FE browser. If this is done, the modified value override the hard coded default values (if any).

All parameter names have a prefix CM_ in the GAMS program. They are discussed in four groups below:

```
PARAMETER CM_CO2GTC(REG,COM) 'Conversion factors between user-defined
                    CO2 commodities and GtC'
PARAMETER CM_EXOFORC(ALLYEAR) 'Radiative forcing from exogenous sources'
PARAMETER CM_MAXC (ITEM,ALLYEAR) 'Maximum allowable level of climate
                    quantity'
PARAMETER CM_HISTORY(ALLYEAR,CM_HISTS) 'Calibration values for CO2
                    and forcing'
PARAMETER CM_CONST(*) 'Climate module constants'
```

1. The parameter CM_CO2GTC is needed to convert various emissions into the total CO2-equivalent expressed in Gt of carbon. In a user’s model, the commodities that describe CO2 or other GHG emissions can have different names and units. Even CO2 emissions may have different units in different regions. In addition, in some models the total CO2 emissions could be divided into e.g. energy-related emissions and non-energy-related emissions, which are described by separate commodities.

2. The parameters CM_EXOFORC and CM_MAXC are identical to those described in section 2. CM_EXOFORC is automatically and CM_MAXC optionally interpolated and extrapolated.
3. The parameter CM_HISTORY is used for the calibration values. To improve independence of data from model years, a **year index** is also included in this parameter. The set CM_HISTS includes the quantity types for which the calibration values are to be given:

CO2-ATM	'Mass of CO2 in the atmosphere (in GtC)'
CO2-UP	'Mass of CO2 in the upper ocean layer (in GtC)'
CO2-LO	'Mass of CO2 in the lower ocean layer (in GtC)'
DELTA-ATM	'Temperature change in the surface'
DELTA-LO	'Temperature change in the deep ocean layer'

If the user provides no values for the CM_EXOFORC or CM_HISTORY quantities, hard-coded default values are applied. The defaults for CM_HISTORY are coded in INITMTY.CLI, and the defaults for CM_EXOFORC are coded in RPT_EXT.CLI. The values in CM_HISTORY are automatically densely inter/extrapolated, and the appropriate base year calibration values are picked up and used in the equations.

4. PARAMETER CM_CONST(*) 'Climate module constants'

This parameter contains the time-independent constants used in the Climate Module:

```
PARAMETER CM_CONST(*) /
  GAMMA          4.1
  PHI-UP-AT      0.0495
  PHI-AT-UP      0.0453
  PHI-LO-UP      0.00053
  PHI-UP-LO      0.0146
  LAMBDA         1.41
  CS             2.91
  SIGMA1         0.024
  SIGMA2         0.44
  SIGMA3         0.002
  CO2-PREIND     596.4
/;
```

User-provided values will automatically override the hard-coded default values shown above. A value specified for CS will determine the value for LAMBDA and vice versa.

Internal parameters

These are equivalent to those used in writing the equations and reporting parameters in sections 4 and 5. An additional internal parameter CM_DEFAULT is used to store the default values for the calibration quantities (see INITMTY.CLI).

Reporting parameters

There is a single reporting parameter CM_RESULT(ALLYEAR,ITEM), which contains all the basic results from the Climate Module, identified by different item names. The reporting parameters described in section 5 are included, as shown in Table 7.2.

7.8.2 Climate related Variables

VAR_CLITOT(item, t): A) global annual atmospheric CO2 emissions in period t (in GtC, ***item***=CO2-GTC); B) total radiative forcing (in W/m2, ***item***=FORCING)

VAR_CLIBOX(item, t): A) masses of CO_2 in period t, in the atmosphere, in a quickly mixing reservoir representing the upper level of the ocean and the biosphere, and in deep oceans, respectively (***item*** = CO2-ATM / CO2-UP / CO2-LO). Units: GtC; B) changes in the atmospheric and deep ocean temperature in period t, in ^{o}C (***item*** = DELTA-ATM / DELTA-LO).

7.8.3 Equations

EQ_CLITOT(item, t): Balance for the total CO2 emissions in GtC (***item***=CO2GTC), and balance for the total forcing in (***item***=FORCING);

EQ_CLICONC(item, t): Balances for the concentrations of CO_2 in the reservoir ***item***;

EQ_CLITEMP(item, t): Balances for the temperature changes in the reservoir ***item***;

EQ_CLIMAX(item, y): Constraint for maximum level of climate quantity ***item***.

All equations are equivalent to those described in section 4.

In addition, the mass balance equations can be calibrated for the first period by using three alternative calibration years $B(1)$–1, $m(1)$–1, and $m(1)$. Whenever $D(1)=1$, the first two alternatives are equal. The historical values represent concentrations and temperatures at the end of each year, and the calibrating values are taken exactly from the calibration year chosen. The default calibrating year is $m(1)$–1. The alternative calibration years can be activated by using one of the following two settings in the run-file:

Table 7.2. Climate Module reporting items for TIMES.

Item name	Climate module variable/parameter	Description
CO2-GTC	**VAR_CO2TOT**	Total CO_2 emissions by milestone year (GtC)
CO2-ATM	**VAR_CO2ATM**	Mass of CO2 in the atmosphere (GtC)
CO2-UP	**VAR_CO2UP**	Mass of CO2 in the upper ocean layer (GtC)
CO2-LO	**VAR_CO2LO**	Mass of CO2 in the deep ocean layer (GtC)
DELTA-FORC	**CM_DT_FORC**	Increase in radiative forcing (W/m^2)
DELTA-ATM	**CM_DT_TATM**	Increase in atmospheric temperature (ºC)
DELTA-LO	**CM_DT_TLOW**	Increase in deep ocean temperature (ºC)

```
$SET CM_CALIB B               ! Calibrate at the end of B(1)–1
$SET CM_CALIB M               ! Calibrate at the end of m(1)
```

7.8.4 Example of use

Because all the constants and calibration values have hard-coded default values, the implementation may be tested in a very simple way. Just add into your data specification the conversion factors from CO2 commodities to the amount of CO2 in GtC. For example, if CO2 emissions are described by a commodity named ‘CO2SUM’ in region ‘REG’ and are expressed in Mt CO2, add the following parameter instance to your dd-file (or directly into the run-file):

```
PARAMETER CM_CO2GTC /
REG.CO2SUM  0.0002727273
                                        /;
```

Moreover, because the current implementation of the Climate Module has been implemented as an extension module, the CLI extension module needs to be activated e.g. by adding a CLI argument to the call of initmty.mod in the run file, as follows:

```
$   BATINCLUDE initmty.mod CLI
```

Finally, as described above, the alternative calibration approach (B) may be activated by including the following setting in the run-file:

```
$  SET CM_CALIB B
```

7.8.5 Exporting results to VEDA-BE

For reporting the results of the climate module the following attributes have been added and appear in VEDA_BE:

- CM_RESULT: All basic results, including the levels of variables and reporting quantities (see Table 7.2),
- CM_MAXC_M: Shadow price of maximum CO2 concentration constraint.

Since these attributes are always declared by the model even if the climate module extension CLI is not used, the standard vdd file can be applied in connection with gdx2veda. In the VEDA back-end software, the result attributes may have different names (e.g. VAR_Climate for CM_RESULT and Dual_clic for CM_MAXC_M).

7.9 References for chapter 7

Drouet L., Edwards N.R. and A. Haurie (2004). “Coupling Climate and Economic Models in a Cost-Benefit Framework: A Convex Optimization Approach”. Submitted to *Environmental Modeling and Assessment.*

IPCC. 1996. *Climatic change 1995: The Science of Climate Change.* Intergovernmental Panel on Climate Change, Second Assessment Report, Working Group I. Cambridge : Cambridge University Press. p.572.

IPCC. 2001. *Climatic change 2001: The Scientific Basis.* Intergovernmental Panel on Climate Change, Third Assessment Report, Working Group I. Cambridge : Cambridge University Press. p.944.

Nordhaus, W. D. and J. Boyer. 1999. *Roll the DICE Again: Economic Models of Global Warming*. Yale University, manuscript edition.

Wigley, T.M.L., Solomon, M. and S.C.B. Raper. 1994. *Model for the Assessment of Greenhouse-Gas Induced Climate Change. Version 1.2.* Climate Research Unit, University of East Anglia, UK.